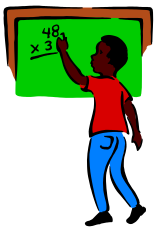
## Fractions

Fractions are representations of parts of a whole … they represent a division process

Fractions can be thought of as a "division in progress."



Example: 1/2 = one half of a whole

Nomenclature: $\frac{\text{Numerator}}{\text{Denominator}}$

Whole number = Counting numbers (0,1,2,3,4,5, 6, etc.)

The denominator is never zero, since any number divided by zero is undefined
(Dividing by zero on a calculator ➔ error message & termination of the calculation)

Proper fraction = numerator smaller than the denominator (example: ¾)
Improper fraction = numerator greater than denominator (example: 4/3)
Mixed number = whole number and fraction (example: 5 ¾)
5 ¾ represents a sum ➔ 5 + ¾

### Converting mixed number to an improper fraction

$$\text{Whole} + \frac{\text{numerator}}{\text{denominator}} = \frac{[(\text{whole}) \times (\text{denominator})] + (\text{numerator})}{\text{denominator}}$$

Example: $5\frac{3}{4} = \frac{(5 \times 4) + 3}{4} = \frac{23}{4}$

### Converting an improper fraction to a mixed number

$$\frac{\text{numerator}}{\text{denominator}} = \text{quotient} + \frac{\text{remainder}}{\text{denominator}}$$

Example: $\frac{29}{6} = 6/\overline{29}$ ➔ 4 5/6

**Equivalent Fractions:**

There are many ways to write the same fraction: termed "equivalent fractions"

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$$

Both numerator & denominator can be multiplied or divided by the same number
Important ➔ whatever is done to the numerator, must also be done to denominator

When no number will simultaneously divide both numerator & denominator, the fraction is considered to be "reduced" or at its "lowest term."

Example: 8/16 reduces to ½ by dividing both numerator & denominator by 8

## Determining if fractions are equivalent

"Cross-multiply" the fractions and compare ➔ fractions are equivalent if products equal

Example: Is 18/21 equivalent to 30/35?

$\frac{18}{21} \times \frac{30}{35}$ 21 x 30 = 630 & 35 x 18 = 630 ➔ the fractions are equivalent

Checking by reduction:

18/21 ➔ dividing both numerator & denominator by 3 ➔ 6/7
30/35 ➔ dividing both numerator & denominator by 5 ➔ 6/7

If cross-multiplication terms are not equal, then the fractions are not equivalent

Example: Is 3/7 equivalent to 2/5?

$\frac{3}{7} \times \frac{2}{5}$ 3 x 5 = 15 & 2 x 7 = 14 ➔ the fractions are not equivalent

The fraction whose numerator gives the largest cross-product is the larger fraction, so
3/7 > 2/5 (where the > symbol means "is greater than")

## Adding Fractions

If denominators are the same, just add the numerators:

$\frac{5}{12} + \frac{1}{12} = \frac{6}{12}$

Reduce the result, if possible
6/12 = ½ (divide both numerator & denominator by 6)

If denominators are different ➔ create equivalent fractions with the same denominator

Conversion Process 1:
Multiply numerator and denominator of first fraction with denominator of the second
Multiply numerator and denominator of second fraction with denominator of the first
Once denominators are the same, add the equivalent fractions

For 3/8 + 5/6

$\frac{3}{8} \rightarrow \frac{(3 \times 6)}{(8 \times 6)} = \frac{18}{48}$ $\frac{5}{6} \rightarrow \frac{(5 \times 8)}{(6 \times 8)} = \frac{40}{48}$

Adding

$\frac{18}{48} + \frac{40}{48} = \frac{58}{48}$

Reduce:

$\frac{(58 / 2)}{(48 / 2)} = \frac{29}{24} = 1\ 5/29$

Conversion Process 2:
If 2 denominators have a common factor, use the smallest (least) common denominator
Once denominators are the same, add the equivalent fractions

For $3/8 + 7/6$ ➔ least common denominator = 24

Converting to fractions with 24 as denominator:

$$\frac{(3 \times 3)}{(8 \times 3)} = \frac{9}{24} \qquad \frac{(7 \times 4)}{(6 \times 4)} = \frac{28}{24}$$

Adding:

$$\frac{9}{24} + \frac{28}{24} = \frac{37}{24} \rightarrow 1\ 13/24$$

**Subtracting Fractions**

Done like addition:
If the denominators are the same, just subtract the numerators:

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

If denominators are different, then convert to equivalent fractions before subtracting

$$\frac{7}{8} - \frac{2}{5}$$

$$\frac{(7 \times 5)}{(8 \times 5)} = \frac{35}{40} \qquad \frac{(2 \times 8)}{(5 \times 8)} = \frac{16}{40} \rightarrow \frac{35}{40} - \frac{16}{40} = \frac{19}{40}$$

**Multiplying Fractions**

Multiple numerators; multiply denominators (no need for common denominator)
Reduce the result

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} \rightarrow \text{(dividing by 2)} \rightarrow \frac{3}{10}$$

**Reciprocals**

The reciprocal (or inverse) of a fraction is created by simply flipping the numbers:
Interchange numerator and denominator

For $\frac{a}{b}$ the reciprocal is $\frac{b}{a}$

the reciprocal of 7/8 is 8/7 ; the reciprocal of 7/3 is 3/7

## Dividing Fractions

Multiply by the reciprocal of the divisor; reduce, if necessary

$\frac{5}{16}$ divided by $\frac{3}{8}$ ➔ $\frac{5}{16} \times \frac{8}{3} = \frac{40}{48}$ ➔ (divide by 4) $\frac{10}{12}$ ➔ (divide by 2) $\frac{5}{6}$

Cannot divide by zero